ÉRETTSÉGI VIZSGA • 2023. október 17.

MATEMATIKA ANGOL NYELVEN

EMELT SZINTŰ ÍRÁSBELI VIZSGA

minden vizsgázó számára

JAVÍTÁSI-ÉRTÉKELÉSI ÚTMUTATÓ

OKTATÁSI HIVATAL

Instructions to examiners

Formal requirements:

1. Mark the paper **legibly, in ink, different in colour** from that used by the candidate.
2. The first of the rectangles next to each problem shows the maximum attainable score on that problem. The **score** given by the examiner is to be entered **into the rectangle** next to this.
3. **If the solution is perfect**, enter maximum score and, with a checkmark, indicate that you have seen each item leading to the solution and consider them correct.
4. If the solution is incomplete or incorrect, please **mark the error** and also indicate the individual **partial scores**. It is also acceptable to indicate the points lost by the candidate if it makes grading the paper easier. After correcting the paper, it must be clear about every part of the solution whether that part is correct, incorrect or unnecessary.
5. Please, **use the following symbols** when correcting the paper:
   - correct calculation: *checkmark*
   - principal error: *double underline*
   - calculation error or other, not principal, error: *single underline*
   - correct calculation with erroneous initial data: *dashed checkmark or crossed checkmark*
   - incomplete reasoning, incomplete list, or other missing part: *missing part symbol*
   - unintelligible part: *question mark* and/or *wave*
6. Do not assess anything written **in pencil**, except for diagrams

Assessment of content:

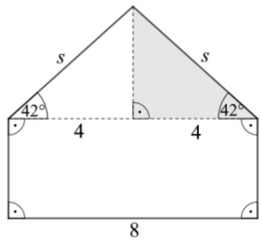
1. The answer key may contain more than one solution for some of the problems. If the **solution given by the candidate is different**, allocate the points by identifying parts of the solution equivalent to those given in the answer key.
2. Subtotals may be **further divided, unless stated otherwise in the answer key**. However, scores awarded must always be whole numbers.
3. If there is a **calculation error** or inaccuracy in the solution, take points off only for the part where the error occurs. If the reasoning remains correct and the error is carried forward while the nature of the problem remains unchanged, points for the rest of the solution must be awarded.
4. **In case of a principal error**, no points should be awarded at all for that section of the solution, not even for steps that are formally correct. (These logical sections of the solutions are separated by double lines in the answer key.) However, if the erroneous information obtained through principal error is carried forward to the next section or the next part of the problem, and it is used there correctly, the maximum score is due for that part, provided that the error has not changed the nature of the task to be completed.
5. Where the answer key shows a **remark** in brackets, the solution should be considered complete without that remark as well.

6. Deduct points for **missing units** only if the missing unit is part of the answer or a unit exchange (without parentheses).
7. If there are more than one different approach to a problem, **assess only the one indicated by the candidate**. Please, mark clearly which attempt was assessed.
8. **Do not give extra points** (i.e. more than the score due for the problem or part of problem).
9. The score given for the solution of a problem, or part of a problem, **may never be negative**.
10. **Do not take points off** for steps or calculations that contain errors but are not actually used by the candidate in the solution of the problem.
11. **The use of calculators** in the reasoning behind a particular solution **may be accepted without further mathematical explanation in case of the following operations**: addition, subtraction, multiplication, division, calculating powers and roots, $n!$, $\binom{n}{k}$, replacing the tables found in the 4-digit Data Booklet (sin, cos, tan, log, and their inverse functions), approximate values of the numbers π and e, finding the solutions of the standard quadratic equation. No further explanation is needed when the calculator is used to find the mean and the standard deviation, as long as the text of the question does not explicitly require the candidate to show detailed work. **In any other cases, results obtained through the use of a calculator are considered as unexplained and points for such results will not be awarded.**
12. Using **diagrams** to justify a solution (e.g. reading data off diagrams by way of direct measurement) is unacceptable.
13. **Probabilities** may also be given in percentage form (unless stated otherwise in the text of the problem).
14. If the text of the problem does not instruct the candidate to round their solution in a particular way, any solution **rounded reasonably and correctly** is acceptable even if it is different from the one given in the answer key.
15. **Assess only four out of the five problems in part II of this paper**. The candidate was requested to indicate in the appropriate square the number of the problem not to be assessed and counted towards their total score. Should there be a solution to that problem, it does not need to be marked. However, if the candidate did not make a selection and neither is their choice clear from the paper, assume automatically that it is the last problem in the examination paper that is not to be assessed.

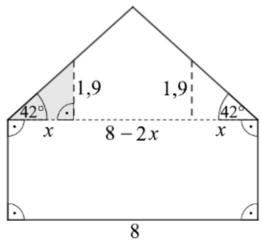
I.

1. a)		
$xy = 12$	1 point	$y = \frac{12}{x}$
(As x and y are positive integers the following (x; y) solutions are possible:) (1; 12), (12; 1), (2; 6), (6; 2), (3; 4), (4; 3).	3 points	*Award 2 points for 5 or 4 correct solutions. 1 point for 3 correct solutions. Deduce 1 point if the candidate gives incorrect answer(s).*
Total:	**4 points**	

1. b)		
$3 \cdot 9^x - 28 \cdot 3^x + 9 = 0$	2 points	
Introduce the new variable $a = 3^x$: $3a^2 - 28a + 9 = 0$ ($a > 0$).	1 point	
$a = 9$ or $a = \frac{1}{3}$	1 point	
In the first case $x = 2$,	1 point	
in the second case $x = -1$.	1 point	
Check by substitution or reference to equivalent steps.	1 point	
Total:	**7 points**	

2. a)		
Let s be the width of one side of one of the rectangles that make up the roof surface. $\cos 42° = \frac{4}{s}$	1 point	
$s \left(= \frac{4}{\cos 42°} \right) \approx 5.4$ m	1 point	
The combined area of the two rectangular parts of the roof: $A = 2 \cdot 5.4 \cdot 8.5 \approx$	1 point	
≈ 92 m^2.	1 point	*Do not award this point if the solution is not rounded or rounded incorrectly.*
Total:	**4 points**	

2. b)		
The volume of the cuboid of the ground floor: $V_f = 8 \cdot 8.5 \cdot 3.2 \approx 218$ m^3.	1 point	
The height of the attic: $m = 4 \cdot \tan 42° \approx 3.6$ m.	1 point	$m = \sqrt{5.4^2 - 4^2} =$ $= \sqrt{13.16} \approx 3.6$ m
The volume of the triangular prism of the attic: $V_t = \frac{8 \cdot 3.6}{2} \cdot 8.5 \approx 122$ m^3.	1 point	
The total volume of the house: $V = V_f + V_t = 218 + 122 = 340$ m^3.	1 point	
Total:	**4 points**	

2. c)		
Let x be the width of the part not considered as living space (see diagram). $\tan 42° = \frac{1.9}{x}$, that makes $x \approx 2.1$ m.	1 point	
The width of living space in the attic is $8 - 2 \cdot 2.1 = 3.8$ m, so the appropriate floor space in the attic is $3.8 \cdot 8.5 \approx 32$ m^2.	1 point	*The floor space with less than 1.9 m in the attic is* $2 \cdot 2.1 \cdot 8.5 \approx 36$ m^2.
The floor space on the ground floor is $8 \cdot 8.5 = 68$ m^2.	1 point	*The full combined floor space of the attic and the ground floor is* $2 \cdot 8 \cdot 8.5 = 136$ m^2.
The total area of the living space in this house is $68 + 32 = 100$ m^2.	1 point	$136 - 36 = 100$ m^2
Total:	**4 points**	

3. a)		
The mean: $\left(\frac{2 + 3 \cdot 6 + 3 \cdot 8}{7} = \frac{44}{7}\right) \approx 6.29$.	1 point	
The standard deviation: $\sqrt{\frac{(-4.29)^2 + 3 \cdot (-0.29)^2 + 3 \cdot 1.71^2}{7}} \approx$	1 point	*Award this point if the candidate uses a calculator to provide the correct answer.*
≈ 1.98.	1 point	
Total:	**3 points**	

3. b) **Solution 1**		
This average means that the sum of the ten scores must be 63, therefore the sum of the scores on the last three games is (63 – 44 =) 19.	1 point	
The range (and the 1-10 scale) means Tomi must either have both a 1 and a 9 (but no 10) or that he does have a 10 (but no 1-s).	1 point	*Award this point if the correct reasoning is reflected only by the solution.*
Case 1: If he has both a 1 and a 9 than the tenth score is (19 – 1 – 9 =) 9. However, this is impossible, as there would be two modes (6 and 8).	1 point	
Case 2: If he has a 10 (but no 1-s) then the sum of the two missing numbers is (19 – 10 =) 9.	1 point	
These two scores may be neither 2 and 7, nor 4 and 5, as that would make two modes (6 and 8).	1 point	
If the missing two scores are 3 and 6 then all conditions are met (the single mode is 6).	1 point	
The last three scores must be 3, 6 and 10 (in some order).	1 point	*The ten scores in increasing order are* 2, 3, 6, 6, 6, 6, 8, 8, 8, 10.
Total:	**7 points**	

3. b) **Solution 2**		
This average means that the sum of the ten scores must be 63, therefore the sum of the scores on the last three games is (63 – 44 =) 19.	1 point	
The single mode of the ten scores must be either 2, or 6, or 8, however, as the sum must be 19, the three missing values must not all be 2-s.	1 point	*Award this point if the correct reasoning is reflected only by the solution.*
Case1 (the mode is 8): Possible solutions: 19 = 8 + 1 + 10 = 8 + 2 + 9 = 8 + 3 + 8 = 8 + 4 + 7.	1 point	
However, none of these would make the correct range (which would be 9, 7, 6, 6, respectively).	1 point	
Case 2 (the mode is 6): Possible solutions: 19 = 6 + 3 + 10 = 6 + 4 + 9 = 6 + 6 + 7.	1 point	
Considering the range (8, 7, 6, respectively), only the first option is possible.	1 point	
The last three scores must be 3, 6 and 10 (in some order).	1 point	
Total:	**7 points**	

3. c)		
Let x be the score on the 11th game. In this case $\frac{63+x}{11}=6.3-\frac{x}{10}$.	1 point	
$630 + 10x = 693 - 11x$	1 point	
$x = 3$ (The score on the 11th game was 3.)	1 point	
Total:	**3 points**	

Note: Award full score if the candidate examines every one of ten different possibilities and gives the correct answer.

4. a)

graph	the function value is positive at *a*	the value of the first derivative is positive at *a*	the value of the second derivative is positive at *a*
I.	*false*	*false*	*false*
II.	*true*	*true*	*false*
III.	*true*	*true*	*true*
IV.	*true*	*false*	*true*

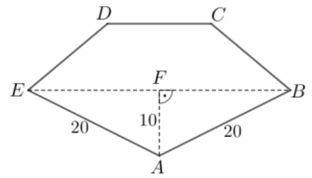
(table above)	5 points*	
The graph of function f is III.	1 point	
Total:	**6 points**	

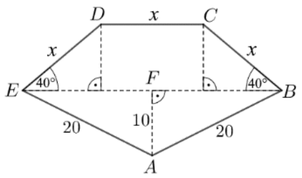
Notes:
*1. Out of the 5 points marked * award 1 for correctly filling the first column, and 2-2 for correctly filling the second and the third columns.*
2. In each column, deduce 1 point from for every incorrectly filled cell, but keep in mind that the score given to that particular column must not be less than 0.

4. b)		
$g'(x) = 2px + q$	1 point	
$g''(x) = 2p$	1 point	
The given conditions state that: $p + q + r = 1$, $2p + q = 2$ and $2p = 4$.	2 points	
So $p = 2$, $q = -2$ and $r = 1$ (thus $g(x)=2x^2-2x+1$, which satisfies all given conditions).	2 points	
Total:	**6 points**	

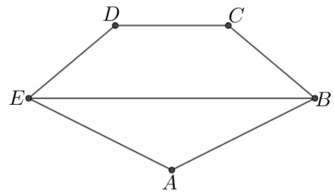
4. c)		
$\int_{-3}^{2}\left(\frac{1}{2}x^2-2x+1\right)dx=\left[\frac{1}{6}x^3-x^2+x\right]_{-3}^{2}=$	1 point	
$=\left(\frac{4}{3}-4+2\right)-\left(-\frac{9}{2}-9-3\right)=-\frac{2}{3}-(-16,5)=\frac{95}{6}$	2 points	
Total:	**3 points**	

II.

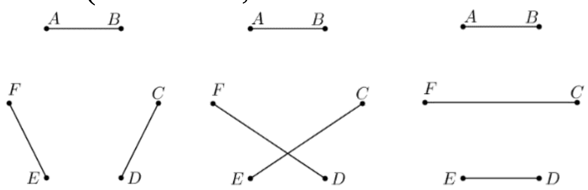
5. a)		
Draw a perpendicular from vertex *A* to diagonal *EB*, this will intersect the diagonal in its midpoint *F*. $AF = 10$ cm.	1 point	
(Triangle *AFB* is half of a regular triangle, so) half of the angle at vertex *A* of the pentagon is 60°.	1 point	$\cos BAF\sphericalangle = 0.5$, *so* $BAF\sphericalangle = 60°$.
Then $BAE\sphericalangle = 120°$ and $ABE\sphericalangle = AEB\sphericalangle = 30°$.	1 point	
(As the angles of the cyclic trapezium are 40°, 40°, 140°, 140°) the angles of the pentagon are 120°, 70°, 140°, 140°, 70°.	1 point	
Total:	**4 points**	

5. b)		
$EB = 2\,BF = 2\sqrt{20^2 - 10^2} = 20\sqrt{3} \approx 34.64$ cm	1 point*	*The area of triangle ABE is equal to the area of a regular triangle of side* 20 cm:
The area of triangle *ABE*: $\frac{EB \cdot FA}{2} = \frac{20\sqrt{3} \cdot 10}{2} = 100\sqrt{3} \approx 173.2$ cm².	1 point*	$\frac{20^2\sqrt{3}}{4} = 100\sqrt{3}$
 Let *x* be the length of side *CD* of the trapezium (in cm-s). Then $EB = x + 2x\cos 40°$.	1 point	
Therefore $34.64 = x + 2x\cos 40° = x(1 + 2\cos 40°)$,	1 point	
$x \approx 13.68$ cm.	1 point	
The height of the trapezium is $x\sin 40° \approx 8.79$ cm,	1 point	
its area is $\left(\frac{34.64 + 13.68}{2} \cdot 8.79\right) \approx 212.4$ cm².	1 point	
The area of the pentagon is (173.2 + 212.4 =) 385.6 cm².	1 point	
Total:	**8 points**	

*Note: Award the points marked * if the candidate uses their answer from part a) to calculate the length of various segments and the area.*

5. c)		
Starting from the degree 2 vertices A, C or D the walk is impossible (as the starting degree 2 vertex must also be the endpoint of the walk, which only allows walking 3, 4 or 5 edges, at most).	1 point	*Award this point if the correct reasoning is reflected only by the solution.*
If (for example) the starting point is B, there are 3 possible options to reach point E, where there are 2 options left to return to E through B.	1 point	
There are $3 \cdot 2 = 6$ options to walk the edges of the graph, starting from B.	1 point	*BCDEBAE*, *BCDEABE*, *BAEBCDE*, *BAEDCBE*, *BEDCBAE*, *BEABCDE*
The same applies when we start from E, so there are a total 12 possible options.	1 point	
Total:	**4 points**	

6. a) Solution 1		
Members of the first team may be selected in $\binom{6}{2} = 15$ different ways.	1 point	
Members of the second team may then be selected in $\binom{4}{2} = 6$ different ways, and the remaining 2 people will form the last team.	1 point	
This is a total $15 \cdot 6 = 90$ possibilities.	1 point	
In each of these 90 options every triplet of teams appears as many different ways as the number of ways three teams can be arranged in various orders (as the order does not matter here),	1 point	
so, in fact, there are $\frac{90}{3!} = 15$ different options to form the three teams.	1 point	
Total:	**5 points**	

6. a) Solution 2		
(Refer to the various names by initials.) If A teams up with, for example, B then there are three different options to select the remaining two teams (CD and EF, CE and DF or CF and DE). 	2 points	
Similarly, for anyone else teaming up with A there are also 3 different options to select the remaining teams. There are 5 possible people to team up with A,	2 points	
so there are, in fact, $5 \cdot 3 = 15$ different options to form the three teams.	1 point	
Total:	**5 points**	

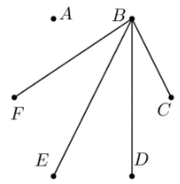
6. a) Solution 3		
Line up the six friends. Let the first and second person in the line make one team, then the 3rd and 4th make another and, finally, the 5th and 6th make the last team.	1 point	
There are 6! (= 720) different lineups.	1 point	
Order within the teams is not considered, which means each actual case has been counted $2 \cdot 2 \cdot 2 = 8$ times.	1 point	
Rearranging the order of the three teams also does not provide a new arrangement. This could be done in 3! (=6) possible ways.	1 point	
There are, in fact, $\frac{6!}{2 \cdot 2 \cdot 2 \cdot 3!} = 15$ different options to form the three teams.	1 point	
Total:	**5 points**	

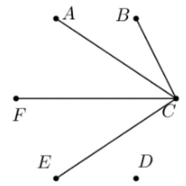
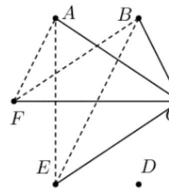
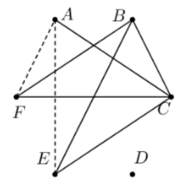
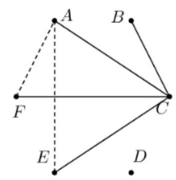
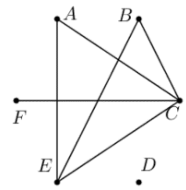
Note: Award full score if the candidate lists all possible arrangements in logical order.

6. b) Solution 1		
Let's first select a partner for Attila. The probability that his partner will be a girl is $\frac{3}{5}$.	1 point	
Then the probability that Csaba's partner will be a girl is $\frac{2}{3}$, which will also guarantee that Emil's partner will be a girl.	1 point	
The probability that each team will consist of one boy and one girl is $\frac{3}{5} \cdot \frac{2}{3} \cdot 1 = \frac{2}{5}$.	2 points	
Total:	**4 points**	

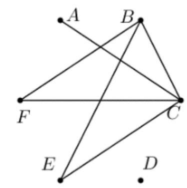
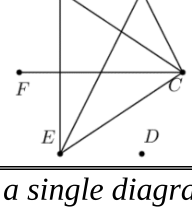
6. b) **Solution 2**		
Assume one member of each team is a boy. Now one girl has to be selected to team up with each boy.	1 point	*Award this point if the correct reasoning is reflected only by the solution.*
This can be done in $3 \cdot 2 \cdot 1 = 6$ ways, so the number of favourable cases is 6.	2 points	*The six options (using initials for names): AB,CD,EF; AB,CF,ED; AD,CB,EF; AD,CF,EB; AF,CB,ED; AF,CD,EB.*
(As the total number of ways to make teams is 15) the probability is $\frac{6}{15} = 0.4$.	1 point	
Total:	**4 points**	

6. b) **Solution 3**		
If teams are formed randomly then those (and only those) cases are unfavourable where there is a team of two girls, one team of two boys and one team with a girl and a boy.	1 point	*Award this point if the correct reasoning is reflected only by the solution.*
The girls-only team may be selected out of the three girls in 3 different ways. For any girls-only team the boys-only team may also be selected out of the three boys in 3 different ways. (The third team will be the one girl and one boy not yet selected.)	1 point	*Let's select one of 3 girls and one of 3 boys to form the mixed-gender team. (In this case, there will be only one way to form the remaining teams in an unfavourable manner.)*
The number of unfavourable cases is $3 \cdot 3 = 9$.	1 point	*This can be done in* $3 \cdot 3 = 9$ *different ways.*
(As the total number of cases is 15) the probability is $1 - \frac{9}{15} = \frac{2}{5}$.	1 point	
Total:	**4 points**	

6. c)		
(Refer to the various names by initials.) As noone competed against their fellow team member, *B*, *C*, *D*, *E* and *F* must each have played 0, 1, 2, 3 or 4 games, which also means every case appeared exactly once.	1 point	
B must not have played 4 games, as there would not be a 0 among the number of games of *C*, *D*, *E* and *F* then.	1 point	

Assume it was *C* who played 4 games. The only one with 0 games must then be *C*’s partner (*D*).	1 point	
Out of the games *B*-*E*, *B*-*F*, *A*-*E*, *A*-*F* select the ones for which the number of games for *B*, *E*, *F* is 1, 2, 3 in whatever order.	1 point*	
If *B* had 3 games, nobody would have 1.	1 point*	
If *B* had 1 game, nobody would have 3.	1 point*	
Which means the only option left for Boróka is to play 2 games (which is possible, too).	1 point	
Total:	**7 points**	

*Note: The points marked * may also be given for the following reasoning:*

Out of the players *B*, *E*, *F* it is impossible for *B* to play 3 games, as that would mean both *E* and *F* must have at least 2 games (with *B* and *C*), so nobody could have played only 1 game.	1 point	
It must be either *E* or *F* then, who has played 3 games. Assume, it was *E* (who must have played *A*, *B* and *C* then).	1 point	
The only one who could have played 1 game is then *F* (with *C*), as both *B*, *C* and *E* must have played at least twice.	1 point	

Note: Award 2 points if the candidate gives the correct answer providing a single diagram based on the story, but does not prove there might not be any other possible solution.

7. a)		
$h(0)=\dfrac{30}{1+59\cdot 0.905^0}=$	1 point	
= 0.5 m is the height of the tree.	1 point	
Total:	**2 points**	

7. b)		
$10=\dfrac{30}{1+59\cdot 0.905^t}$	1 point	
$590\cdot 0.905^t=20$ $0.905^t=\dfrac{2}{59}$	1 point	$0.905^t\approx 0.034$
$t=\log_{0.905}\dfrac{2}{59}$	1 point	$t\approx\dfrac{\log 0.034}{\log 0.905}$
$t\approx 33.9$	1 point	
About 34 years after the beginning of the observation will the tree be 10 m tall.	1 point	
Total:	**5 points**	

7. c)		
The sequence $\{0.905^n\}$ is convergent and its limit is 0.	1 point	*Award this point if the correct reasoning is reflected only by the solution.*
The sequence $\{a_n\}$ is, therefore, also convergent and $\lim\limits_{n\to\infty}\{a_n\}=\dfrac{30}{1+59\cdot 0}=\dfrac{30}{1}=30.$	2 points	
Total:	**3 points**	

7. d)		
Let x be the length of the section of fence that costs 5 thousand Ft/m and let y be the length of the section that costs 10 thousand Ft/m ($x > 0$, $y > 0$). The full building cost is then $5x + 10y = 400$ (thousand Ft). From this $y = 40 - 0.5x$.	1 point	$x=80-2y$
The area of the rectangular plot of land (in m^2-s): $xy=x(40-0.5x)=40x-0.5x^2$.	1 point	$xy=(80-2y)\,y=$ $=80y-2y^2$
$40x-0.5x^2=-0.5(x-40)^2+800$	1 point*	$80y-2y^2=$ $=-2(y-20)^2+800$
This has a maximum at $x = 40$,	1 point*	*This has a maximum at* $y = 20$,
when $y = 20$.	1 point	*when* $x = 40$.

The rectangular plot that has the maximum area while it is still within the budget has 40 m of fencing on the side that costs 5 thousand Ft/m, and 20 m of fencing on the side that costs 10 thousand Ft/m. (The area of the plot is then 800 m^2.)	1 point	
Total:	**6 points**	

*The points marked * may also be given for the following reasoning:*

$f:]0; 80[\to \mathbf{R}$; $f(x) = 40x - 0.5x^2$. A necessary condition for extremes is $f'(x) = 40 - x = 0$.	1 point	
This gives $x = 40$ and the derivative turns from positive to negative, so this is a (global) maximum of f.	1 point	

or

$xy = x(40 - 0.5x) = 0.5 \cdot x(80 - x)$. Apply the inequality about the arithmetic and geometric means for the factors of the product $x(80 - x)$: $x(80-x) \le \left(\frac{x+80-x}{2}\right)^2 = 1600.$	1 point	
Equation is only allowed when $x = 80 - x$, i.e. when $x = 40$. This must be where f has its maximum.	1 point	

8. a)		
Event E_0 occurs if and only if both passages from section A are closed.	1 point	
The probability of this is $(1 - p)^2$.	1 point	
$(1-p)^2 = 1 - 2p + p^2$, indeed.	1 point	
Total:	**3 points**	

8. b)		
$1 - 2p + p^2 = (1-p)^2$, so $(1-p)^2 \le 0{,}01$.	1 point	$1 - 2p + p^2 \le 0.01$ $p^2 - 2p + 0.99 \le 0$
$p < 1$ and so $0 < 1 - p \le \sqrt{0.01} = 0.1$.	1 point	*The real roots of* $p^2 - 2p + 0.99 = 0$ *are 0.9 and 1.1.*
The probability of event E_0 will be 0.01 at most if $0.9 \le p\ (<1)$.	1 point	
Total:	**3 points**	

8. c)		
Event E_1 occurs if and only if one of the passages from section A is open, the other one is closed and also the passage between B and C is closed.	1 point	
The probability of this is $\binom{2}{1}p(1-p)\cdot(1-p)=$	1 point	
$=2p-4p^2+2p^3$, indeed.	1 point	
Event E_2 occurs if and only if two or more (i.e. 2 or 3) passages are open. The probability of this is $p^3+\binom{3}{2}p^2(1-p)=$	1 point*	
$=p^3+3p^2-3p^3=3p^2-2p^3$, indeed.	1 point*	
Total:	**5 points**	

*Note: The points marked * may also be given for the following reasoning:*

(The events E_0, E_1 and E_2 are mutually exclusive and cover the entire event space, so) $P(E_2) = 1 - P(E_0) - P(E_1) =$	1 point	
$=1-(1-2p+p^2)-(2p-4p^2+2p^3)=3p^2-2p^3$	1 point	

8. d)		
(The function $f:]0; 1[\to \mathbf{R}$; $f(p)=2p-4p^2+2p^3$ will assume its maximum where its derivative is zero.) $f'(p)=2-8p+6p^2=0$	1 point	
$p = 1$ or $p=\frac{1}{3}$	1 point	
$p = 1$ is not correct (as $p < 1$).	1 point	
In case of $p=\frac{1}{3}$, $f''\left(\frac{1}{3}\right)=-8+4=-4<0$, so it is a maximum of the function. (A global maximum on the interval]0; 1[.)	1 point	*At $p=\frac{1}{3}$ the function switches from increasing to decreasing, making this a maximum.*
The probability is then: $\left(2\cdot\frac{1}{3}-4\cdot\frac{1}{9}+2\cdot\frac{1}{27}=\right)\frac{8}{27}$ (≈ 0.296).	1 point	
Total:	**5 points**	

9. a)		
(There exist a total $\binom{5}{2}=10$ such products of two factors.) $2\cdot4,\ 2\cdot6,\ 2\cdot8,\ 2\cdot10,$ $4\cdot6,\ 4\cdot8,\ 4\cdot10,$ $6\cdot8,\ 6\cdot10,$ $8\cdot10.$	3 points	
The sum of these is (8 + 12 + 16 + 20 + 24 + 32 + 40 + 48 + 60 + 80 =) 340.	1 point	
Total:	**4 points**	

9. b)		
$S_{k+1}=S_k+1\cdot(k+1)+2\cdot(k+1)+...+k\cdot(k+1)$	2 points	
$S_{k+1}=S_k+(1+2+...+k)\cdot(k+1)=$	1 point	
$=S_k+\frac{k(k+1)}{2}\cdot(k+1)=S_k+\frac{k(k+1)^2}{2}$, indeed.	1 point	
Total:	**4 points**	

9. c) Solution 1		
Proof by induction: when n = 2, $S_2=1\cdot2=2=\frac{1\cdot2\cdot3\cdot8}{24}=\frac{48}{24}$ is true.	1 point	
It is enough to prove that, once the statement is true for any particular value of k ($k\in\mathbf{N}$, $k\geq2$), it is also true for the value k + 1, i.e. $S_{k+1}=\frac{k(k+1)(k+2)(3k+5)}{24}$.	1 point	*Award this point if the correct reasoning is reflected only by the solution.*
From part b) $S_{k+1}=S_k+\frac{k(k+1)^2}{2}=$ $=\frac{(k-1)k(k+1)(3k+2)}{24}+\frac{k(k+1)^2}{2}$.	1 point	
factoring $k(k+1)$: $S_{k+1}=k(k+1)\cdot\frac{(k-1)(3k+2)+12(k+1)}{24}=$	1 point	
$=k(k+1)\cdot\frac{3k^2+11k+10}{24}$.	1 point	
As $3k^2+11k+10=(k+2)(3k+5)$,	2 points	
$S_{k+1}=\frac{k(k+1)(k+2)(3k+5)}{24}$, which means the original statement is true.	1 point	
Total:	**8 points**	

9. c) Solution 2		
(n is a given positive integer, $n \geq 2$.) $(1+2+3+...+n)^2 =$ $=(1^2+2^2+...+n^2)+2\cdot(1\cdot 2+1\cdot 3+...+(n-1)\cdot n)$.	1 point	
From the above: $2\cdot(1\cdot 2+1\cdot 3+...+(n-1)\cdot n)=$ $=(1+2+...+n)^2-(1^2+2^2+...+n^2)$.	1 point	
As $(1+2+...+n)^2=\left(\frac{n(n+1)}{2}\right)^2$	1 point	
and $1^2+2^2+...+n^2=\frac{n(n+1)(2n+1)}{6}$,	1 point	
so $2\cdot(1\cdot 2+1\cdot 3+...+(n-1)\cdot n)=$ $=\frac{n^2(n+1)^2}{4}-\frac{n(n+1)(2n+1)}{6}=$	1 point	
$=n(n+1)\left(\frac{n(n+1)}{4}-\frac{2n+1}{6}\right)=n(n+1)\cdot\frac{3n^2-n-2}{12}$.	1 point	
$3n^2-n-2=(3n+2)(n-1)$, so $2\cdot(1\cdot 2+1\cdot 3+...+(n-1)\cdot n)=\frac{(n-1)n(n+1)(3n+2)}{12}$.	1 point	
On division by 2 the statement to be proven is obtained.	1 point	
Total:	**8 points**	